[Volume 46, Part 14](https://www.sciencedirect.com/journal/materials-today-proceedings/vol/46/part/P14), 2021, Pages 6161-6167



**Effect of corrugation and frequency parameters due to quasi harmonic-waves on a corrugated interface of two different nematic elastomers**

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**Abstract**

The reflection and refraction phenomena due to incident quasi harmonic-wave at an irregular interface of two different nematic [elastomer](https://www.sciencedirect.com/topics/materials-science/elastomer) half-spaces has been investigated. The expressions of phase velocity for incident, reflected and refracted *qSH*-waves are obtained which depends on the angle of propagation. Besides the regular waves, there exist an irregularly reflected and refracted waves due to undulated nature of the interface whose nth spectrum is related through the spectrum theorem. Using Rayleigh’s approximation method, we assume that the amplitude and slope of corrugation are both small. The expressions of the amplitude ratios corresponding to the reflected and refracted waves are derived by using appropriate boundary conditions -continuity of (i) displacements and (ii) tractions at the irregular interface. These ratios are found to be functions of elastic constants, angle of incidence, relaxation times, corrugation and frequency parameters. The energy distribution and hence energy ratios of various reflected and refracted waves are also obtained. The amplitude and energy ratios are computed numerically for a particular model �(x)=������ and are plotted in graphs and discussed the effects of corrugation and frequency parameter. In the amplitude and energy ratios, irregularities occur between the angles 780 and 850 due to the existence of critical and moreover at 830 irregularities show dominant behavior. We concluded that (i) the ratios due to irregular waves are comparatively small to that due to regular waves. (ii) The amplitude and energy ratios corresponding to irregular waves show linear and non-linear increase respectively with the increase of corrugation parameter. (iii) Theoretically and numerically, the ratios corresponding to the regular waves are independent of corrugation and frequency parameters. (iv) The sum of energy ratios is approximately unity at each value of incident angle which ensures the law of conservation of energy at the interface.

**Introduction**

NEs are rubber-like liquid crystalline materials made up of cross-linking of nematic crystalline molecules called mesogens either incorporated into the main chain or pendant from them. They simultaneously combine the elastic properties of ordinary rubbers with the anisotropy of liquid crystals. The liquid crystals within the nematic elastomer have a temperature-dependent interaction. At low temperature, the mesogens tend to align themselves and give rise to a local nematic orientational order described by a director. On the other hand, the mesogens are randomly oriented at high temperature and the material is isotropic [1], [2]. These materials display many unusual mechanical properties including the formation of fine-scale microstructures and fine-scale wrinkles. The peculiar property of such material is the presence of considerable long macromolecules with rare intermolecular transversal bonds [3], [4]. Besides their unusual static properties, an internal relaxation of the nematic director is responsible for its dynamic soft elasticity. Of particular interest is their soft matter property which make NEs an increasingly subject of numerous studies in the fields of microelectronics, biomechanics, nanomechanics. Moreover, they also have novel responses to external stimuli such as electric fields, temperature and light [5], [6]. Kielczynski and Pajewski [7] analyzed the validity of an approximate expression for the reflection coefficient due to obliquely incident SH-plane wave at a plane interface between an elastic solid and a viscoelastic liquid. Bladon et al. [8] showed that the nematic director can experience a barrier to its rotation up to a certain critical strain. Kupfer et al. [9] synthesized a series of liquid single-crystal elastomers with different cross-linking densities. Verwey et al. [10] studied the elastic and orientational response of a uniform nematic elastomer subjected to an extension perpendicular to its director. Selinger et al. [11] showed that the nematic order parameter and the induced strain vary smoothly across the isotropic-nematic transition. Anderson et al. [12], Chattopadhyay et al. [13], Singh et al. [14], Matteis [15], Warner and Terentjev [16], Fradkin et al. [17], Clarke et al. [18] and Terentjev et al. [19] also discussed different problems in nematic elastomers.

The theories of wave propagation from discontinuities are of great practical importance in the field of earthquake engineering, geophysics and seismology. Different layers in the Earth's crust has discontinuities between them which cannot be assumed as a perfect plane, rather irregular to some extent. These irregular interfaces influenced the reflection and refraction phenomena of elastic waves to a very large extent. It was Rayleigh [20] who first attempted the problem of the reflection and refraction of waves at irregular boundary surface. Singh [21] discussed the plane harmonic wave propagation in anisotropic nematic elastomers and obtained the phase velocity and attenuation coefficients. Zakharov [22] investigated localized waves near the stress-free surface or the free edge of a solid with a thin nematic coating. Singh and Tomar [23] attempted the problem of reflection and refraction phenomena due to incident elastic waves at a corrugated interface between two dissimilar fibre-reinforced elastic half-spaces. Singh [24] investigated the propagation of plane shear wave at a corrugated interface between elastic solid and viscoelastic liquid half-spaces. Plucinsky and Bhattacharya [25] explored the response of taut and appreciably stressed sheets made of nematic elastomer and showed that nematic elastomer sheets can suppress wrinkling by modifying the expected state of stress through the formation of microstructure. Different methods have been adopted to deal with the scattering of elastic waves from irregular interfaces, i.e., Asano [26], Abubakar [27], Kaur et al. [28] and Tomar and Saini [29]. Yang et al. [30] studied the characteristic equations for Rayleigh wave propagation in NEs based on viscoelastic theory at low frequency limit and analyzed the dispersion and attenuation properties numerically. Zhao and Liu [31] studied the problem of transverse wave dispersion in an NE beam by considering anisotropy and viscoelasticity in the low frequency limit and discussed the influences of anisotropic parameter, director rotation and rubber relaxation times on the wave dispersion in an NE beam.

The objective of the present study is to highlight the influence of corrugation and frequency parameters in the reflection and refraction coefficients as well as in the energy ratios of the irregular waves which are caused by the corrugated interface of the medium. In case of plane interface only the regular wave exists, but the present study involves the case of corrugated interface where there exists irregularly reflected and refracted waves. The propagation of elastic waves and their reflection and refraction from various types of surfaces/interfaces are of great importance in the field of seismology, earthquake engineering and also in signal processing. The techniques of seismic wave propagation are tools for investigating the internal structure of the earth as well as for exploration of valuable materials such as oil, water, chemical, *etc*. beneath the earth surface. The nature of the earth materials play an important role in changing the characteristics of the seismic signals recorded on the surface of the earth. The layers in the Earth's crust have discontinuities which greatly affect the reflection and refraction phenomena and hence it is important to take into account the problems related to the effects of irregular interfaces. In this article with the help of Rayleigh's method of approximation, the problem of incident quasi shear harmonic-wave at irregular interface between two different nematic elastomer half-spaces have been investigated. The expressions of amplitude and energy ratios corresponding to the reflected and refracted waves are obtained in the closed form and computed numerically for a specific model for different values of frequency and corrugation parameters. The results of Ben-Menahem and Singh [32], Singh [33] and Asano [26] of the relevant problems are recovered from the present work.

**Section snippets**

**Constitutive equation and wave propagation**

Following [16], the elastic potential energy density in nematic solid is given by∊∊∊∊∊�=�1�.∊.�2+2�2����.∊.�+�3���2+2�4�×∊×�2+4�5(�×(∊.�))2+12�1(n×Θ)2+D2n.∊.(n×Θ)Here, we restricted the Frank elastic energy only for uniform director rotations of the NEs represented by (�×��), Θ=Ω-(�×��) is an independent rotational variable, ∊∊ij=�ij-����ij/3,(�,�=1,2,3) is the traceless part of of linear symmetric strain, �� are elastic constants with D1 and D2 are coupling constants. This equation describes two

**Boundary condition and solution**

The components of displacement and traction are continuous at the irregular interface, �=�(�). Mathematically, these conditions at may be written as�=�′,�23-�21�′=�′23-�′21�′,

Using Eq. (2.3), the boundary condition reduces to1+�����44�����-�′�66����=1+���′�(�′44���′��-�′�′66��′��)

Inserting Eqs. (2.6), (2.8), (2.9) into Eqs. (2.9), (3.1), we get∑�=01������3�+∑�=1∞�1�±�±��������3±1�=�2����3(2)+∑�=1∞�2�±�±��������3±(2�)∑�=01��(�44��3�-�66�1��′)����3(�)+∑�=1∞�1�±(�44��3±1�-�66�1±1��′)�±��������3±1�=

**Distribution of energy**

The energy due to the incident *qSH*-wave is distributed to regularly and irregularly reflected and refracted waves. The rate of transmission of energy per unit area is given by [34]

℘∗=<�23.�̇> (4.1)

The energy due to the incident *qSH*-wave is given by�inc=�0��02exp2��t+�10x+�30z,

where�0=1+�����44��30Similarly, the energy due to reflected and refracted *qSH-*wave are given by��=�����2exp{2�(�t+�1�x+�3�z)}+∑�=1∞�mn±�(�mn±)2exp{2�(�t+�1±��x+�3±(��)z)},�=1,2

The expressions of �� and �mn± are given by ��=

**Special case**

If the irregular interface is represented by only cosine term �=�cos��, then coefficient of expansion becomes�±�=�2���=10���≠1.Then, using these values in Eq. (3.9), the amplitude ratios reduces to�1±=�2±-�0�′44��3±(21)�1±�44��3±(11)-�0�′44��3±(21),�1±=�2±-�44��3±(11)�1±�44��3±(11)-�0�′44��3±(21)where�1±=��2-�30-�1�0�31+�2�0�32,�2±=��2(-�0±-�1�0�1±+�2�0�2±)�0±=�44��3(0)�3(0)±�66��10,�1±=�44��3(1)�3(1)±�66��11,�2±=�0(�′44��3(2)�3(2)±�′66��12)

In the normal incidence, �0=00, we obtain that ����11+=

**Particular case**

* (a)

When the two nematic half-spaces Ω and Ω' reduce to isotropic half-spaces, we have

�1=�2=0,�11=�33=�+2�,�13=�,�44=�44�=�66=�,�1=�4=�5=12�,�′1=�′2=0,c′11=c′33=�′+2�′,c′13=�′,c′44=c′44R=c′66=�′,C′1=C′4=C′5=12�′.

The phase velocity corresponding to the transverse wave becomes �22=��,�′22=�′�′ which are the results of classical elasticity [34]. The amplitude ratios of the reflected and refracted waves are given by Eqs. (3.4), (3.6) with the following modified values�′0=�′�,�0±=�3(0)�3(0)±���10,�1±=�3(

**Numerical computation and discussion**

We will evaluate the angles of reflected and refracted waves through Snell's law. Introducing an apparent velocity �� related with dimensionless velocity by �-=���=��1� into Eq. (2.7) we get�-2=1+�����-66+�-44��2,where�=�3�1,�=�44�,�-ij=�ij�44,Transforming this equation with �=1�=�1�3, we get�2=1+�����-44��-2-1+�����-66

Eq. (7.2) give rises the reflected angle �1=tan-1(�) for a given �0. Similarly, in the half-space Ω′, the angle of refracted *qSH*-wave is obtained as �2=tan-1�′. For numerical

**Concluding remarks**

The amplitude and energy ratios due to regular and irregular waves are computed numerically for a specific model, �=������ and we may conclude with the following remarks:

* (i)

The angles corresponding to the reflected and refracted waves increase with the increase of the angle of incidence.

* (ii)

All the amplitude and energy ratios corresponding to irregular waves are functions of the angle of incidence, elastic constants, coupling constants, the characteristic time of rubber relaxation, the director

**Declaration of Competing Interest**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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