ON A TYPE OF SEMI-GENERALIZED RECURRENT P-SASAKIAN MANIFOLDS

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ON A TYPE OF SEMI-GENERALIZED RECURRENT P-SASAKIAN MANIFOLDS

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Abstract. In the present paper we study some geometrical properties of semi-generalized recurrent P-Sasakian manifolds.
keywords: Semi-generalized recurrent manifold, P-Sasakian manifolds, Concircular curvature tensor, Einstein manifold, M-projective curvature tensor.

1. Introduction

The idea of recurrent manifolds was introduced by A.G. Walker in 1950 [16]. On the other hand, De and Guha [3] introduced generalized recurrent manifold with the non-zero 1-form A and another non-zero associated 1-form B. Such a manifold has been denoted by GK_n. If the associated 1-form B becomes zero, then the manifold GK_n reduces to a recurrent manifold introduced by Ruse [12] which is denoted by K_n. In 1977, Adati and Matsumoto [1] defined P-Sasakian and Special Para Sasakian manifolds, which are special classes of an almost para-contact manifold introduced by Sato [13]. Para Sasakian manifolds have been studied by De and Pathak [4], Matsumoto et.al. [8], Matsumoto [9], Shukla and Shukla [14], Singh [15], De and Sarkar [5] and many others.

A Riemannian manifold (M^n, g) is called a semi-generalized recurrent manifold if its curvature tensor R satisfies the condition

$$(\nabla_X R)(Y,Z)W = A(X)R(Y,Z)W + B(X)g(Z,W)Y, \tag{1.1}$$

where A and B are two 1-forms, B is non-zero, P_1 and P_2 are two vector fields such that

$$g(X,P_1) = A(X), \qquad g(X,P_2) = B(X), \tag{1.2}$$

for any vector field X and ∇ denotes the operator of covariant differentiation with respect to the metric g.

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Generalizing the notion of recurrency the author Khan [6] introduced the notion of generalized recurrent Sasakian manifold. In the paper [11], B. Prasad introduced the notion of semi-generalized recurrent manifold and obtained some interesting results. Recently Rajesh Kumar et.al. [7] studied semi generalized recurrent LP-Sasakian manifolds. Motivated by the above studies, in this paper we extend the study of semi-generalized recurrent to Para-Sasakian manifolds. The paper is organized as follows: Section 2, consist the basic definitions of P-Sasakian and Einstein manifolds. In Section 3, we studied semi-generalized recurrent P-Sasakian manifolds. In section 4, we prove that a semi-generalized ϕ-recurrent P-Sasakian manifold is an Einstein manifold. Section 5 is devoted to the study of semi-generalized concircular ϕ-recurrent P-Sasakian manifolds. Section 6 is about the study of M-projective ϕ-recurrent P-Sasakian manifolds respectively. In the last section we studied the three-dimensional locally semi-generalized ϕ-recurrent P-Sasakian manifolds.

2. Introduction

An n-dimensional differentiable manifold M^n is a Para-Sasakian (briefly P-Sasakian) manifold if it admits a (1,1) tensor field ϕ, a contravariant vector field ξ, a covariant vector field η, and a Riemannian metric g, which satisfy

$$\phi^2 X = X - \eta(X)\xi, \quad g(X,\xi) = \eta(X), \quad \phi\xi = 0, \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X,Y) - \eta(X)\eta(Y), \tag{2.2}$$

$$(\nabla_X \phi)Y = -g(X,Y)\xi - \eta(Y)X + 2\eta(X)\eta(Y)\xi, \tag{2.3}$$

$$\nabla_X \xi = \phi X, \tag{2.4}$$

$$(\nabla_X \eta)(Y) = g(\phi X, Y) = g(\phi Y, X), \tag{2.5}$$

for any vector fields X and Y, where ∇ denotes covariant differentiation with respect to g ([1],[13]).

It can be seen that in a P-Sasakian manifold M^n with the structure (ϕ, ξ, η, g), the following relations hold:

$$\eta(\xi) = 1, \quad \eta(\phi X) = 0, \tag{2.6}$$

$$rank(\phi) = (n-1). \tag{2.7}$$

Further in a P-Sasakian manifold the following relations also hold:

$$\eta(R(X,Y)Z) = g(X,Z)\eta(Y) - g(Y,Z)\eta(X), \tag{2.8}$$

$$R(X,Y)\xi = \eta(X)Y - \eta(Y)X, \tag{2.9}$$

$$R(\xi,X)Y = \eta(Y)X - g(X,Y)\xi, \tag{2.10}$$

$$R(\xi,X)\xi = X - \eta(X)\xi, \tag{2.11}$$

$$Q\xi = -(n-1)\xi, \tag{2.12}$$

$$S(X,\xi) = -(n-1)\eta(X), \tag{2.13}$$

$$S(\phi X,\phi Y) = S(X,Y) + (n-1)\eta(X)\eta(Y), \tag{2.14}$$

for any vector fields X, Y, Z, where R and S are the Riemannian curvature tensor and Ricci tensor of the manifold respectively.
A P-Sasakian manifold M^n is said to be Einstein if the Ricci tensor S is of the form

$$S(X,Y) = \lambda g(X,Y), \tag{2.15}$$

where λ is a constant.

3. Semi-Generalized Recurrent P-Sasakian manifolds

Definition 3.1. A Riemannian manifold (M^n, g) is semi-generalized Ricci recurrent manifold ([3],[2]) if

$$(\nabla_X S)(Y,Z) = A(X)S(Y,Z) + nB(X)g(Y,Z). \tag{3.1}$$

Theorem 3.1. *The scalar curvature r of a semi-generalized recurrent P-Sasakian manifold is related in terms of contact forms $\eta(P_1)$and $\eta(P_2)$ as given by*

$$r = -\frac{1}{\eta(P_1)}\left[(n^2+2)\eta(P_2) + 2(n-1)\eta(P_1)\right].$$

Proof. Permutting equation (1.1) twice with respect to X, Y, Z; adding the three equations and using Bianchi's second identity, we have

$$\begin{aligned} & A(X)R(Y,Z)W + B(X)g(Z,W)Y + A(Y)R(Z,X)W \\ + \quad & B(Y)g(X,W)Z + A(Z)R(X,Y)W + B(Z)g(Y,W)X = 0. \end{aligned} \tag{3.2}$$

Contracting (3.2) with respect to Y, we get

$$\begin{aligned} & A(X)S(Z,W) + nB(X)g(Z,W) - g(R(Z,X)P_1, W) \\ + \quad & B(Z)g(X,W) - A(Z)S(X,W) + B(Z)g(X,W) = 0. \end{aligned} \tag{3.3}$$

In view of $S(Y,Z) = g(QY,Z)$, the equation (3.3) reduces to

$$\begin{aligned} &A(X)g(QZ,W) + nB(X)g(Z,W) - g(R(Z,X)P_1,W) \\ &+B(Z)g(X,W) - A(Z)g(QX,W) + B(Z)g(X,W) = 0. \end{aligned} \tag{3.4}$$

Factoring off W, we get from (3.4)

$$\begin{aligned} A(X)QZ &+ nB(X)Z - R(Z,X)P_1 \\ &+ B(Z)X - A(Z)QX + B(Z)X = 0. \end{aligned} \tag{3.5}$$

Contracting (3.5) with respect to Z, we get

$$A(X)r + (n^2 + 2)B(X) - 2S(X,P_1) = 0. \tag{3.6}$$

Putting $X = \xi$ in the equation (3.6) and using the equations (1.2) and (2.13), we get

$$r = -\frac{1}{\eta(P_1)}\left[(n^2 + 2)\eta(P_2) + 2(n-1)\eta(P_1)\right].$$

This completes the proof. □

Theorem 3.2. *In a semi-generalized Ricci-recurrent P-Sasakian manifold, the 1-form A and B are related as*

$$-(n-1)A(X) + nB(X) = 0.$$

Proof. Taking $Z = \xi$ in (3.1), we have

$$(\nabla_X S)(Y,\xi) = A(X)S(Y,\xi) + nB(X)g(Y,\xi). \tag{3.7}$$

The left hand side of (3.7), clearly can be written in the form

$$(\nabla_X S)(Y,\xi) = \nabla_X S(Y,\xi) - S(\nabla_X Y,\xi) - S(Y,\nabla_X \xi),$$

which in view of (2.4), (2.5) and (2.13) gives

$$-(n-1)g(Y,\phi X) - S(Y,\phi X).$$

While the right hand side of (3.7) equals

$$A(X)S(Y,\xi) + nB(X)g(Y,\xi) = -(n-1)A(X)\eta(Y) + nB(X)\eta(Y).$$

Hence,

$$-(n-1)g(Y,\phi X) - S(Y,\phi X) = -(n-1)A(X)\eta(Y) + nB(X)\eta(Y). \tag{3.8}$$

Putting $Y = \xi$ in (3.8) and then using (2.1), (2.6) and (2.13), we get

$$-(n-1)\eta(\phi X) + (n-1)\eta(\phi X) = -(n-1)A(X) + nB(X),$$

Or,

$$-(n-1)A(X) + nB(X) = 0. \tag{3.9}$$

□

Theorem 3.3. *If a semi-generalized Ricci-recurrent P-Sasakian manifold is an Einstein manifold then 1-forms A and B are related as* $\lambda A(Y) + nB(Y) = 0$.

Proof. For an Einstein manifold, we have $S(Y,Z)=\lambda g(Y,Z)$ and $(\nabla_U S)=0$, where λ is constant.
Hence from (3.6) we have

$$\begin{aligned}[\lambda A(X) + nB(X)]\, g(Y,Z) &+ [\lambda A(Y) + nB(Y)]\, g(Z,X)\\ +[\lambda A(Z) + nB(Z)]\, g(X,Y) &= 0.\end{aligned} \tag{3.10}$$

Replacing Z by ξ in (3.10) and using (1.2) and (2.1), we have

$$\begin{aligned}[\lambda A(X) + nB(X)]\, \eta(Y) &+ [\lambda A(Y) + nB(Y)]\, \eta(X)\\ +[\lambda \eta(P_1) + n\eta(P_2)]\, g(X,Y) &= 0.\end{aligned} \tag{3.11}$$

Again, taking X=Y=ξ in (3.11) and using (1.2), (2.1) and (2.6), we have

$$[\lambda \eta(P_1) + n\eta(P_2)] = 0. \tag{3.12}$$

Using (1.2) and (2.1) in the above relation, it follows that

$$\lambda A(Y) + nB(Y) = 0.$$

□

4. Semi-Generalized ϕ-Recurrent P-Sasakian manifolds

Definition 4.1. A P-Sasakian manifold (M^n, g) is called semi-generalized ϕ recurrent if its curvature tensor R satisfies the condition

$$\phi^2((\nabla_W R)(X,Y)Z) = A(W)R(X,Y)Z + B(W)g(Y,Z)X, \tag{4.1}$$

where A and B are two 1-forms, B is non-zero and these are defined by

$$A(W) = g(W, P_1), \quad B(W) = g(W, P_2) \tag{4.2}$$

and P_1 and P_2 are vector fields associated with 1-forms A and B, respectively.

Theorem 4.1. *A semi generalized ϕ-recurrent P-Sasakian manifold (M^n, g) is an Einstein manifold and moreover; the 1-forms A and B are related as* $(n-1)A(W) = nB(W)$.

Proof. Let us consider a semi-generalized ϕ-recurrent P-Sasakian manifold. Then by virtue of (2.1) and (4.1) we have

$$\begin{aligned}&(\nabla_W R)(X,Y)Z - \eta((\nabla_W R)(X,Y)Z)\xi\\ &= A(W)R(X,Y)Z + B(W)g(Y,Z)X.\end{aligned} \tag{4.3}$$

From which it follows that

$$g((\nabla_W R)(X,Y)Z,U) - \eta((\nabla_W R)(X,Y)Z)\eta(U)$$
$$(4.4) \qquad = A(W)g(R(X,Y)Z,U) + B(W)g(Y,Z)g(X,U).$$

Let $\{e_i\}, i = 1, 2, ...n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $X = U = e_i$ in (4.4) and taking summation over i, $1 \leq i \leq n$, we get

$$(\nabla_W S)(Y,Z) - \sum_{i=1}^{n} \eta((\nabla_W R)(e_i,Y)Z)\eta(e_i)$$
$$(4.5) \qquad = A(W)S(Y,Z) + nB(W)g(Y,Z).$$

The second term of left hand side of (4.5) by putting $Z = \xi$ takes the form $g((\nabla_W R)(e_i,Y)\xi,\xi)$, which is zero in this case. So, by replacing Z by ξ in (4.5) and using (2.13), we get

$$(4.6) \qquad (\nabla_W S)(Y,\xi) = -(n-1)A(W)\eta(Y) + nB(W)\eta(Y).$$

We know that

$$(\nabla_W S)(Y,\xi) = \nabla_W S(Y,\xi) - S(\nabla_W Y,\xi) - S(Y,\nabla_W \xi),$$

using (2.4), (2.5) and (2.13) in the above relation, it follows

$$(4.7) \qquad (\nabla_W S)(Y,\xi) = -(n-1)g(\phi W,Y) - S(\phi W,Y).$$

From (4.6) and (4.7) we obtain

$$(4.8) \qquad -(n-1)g(\phi W,Y) - S(\phi W,Y) = -(n-1)A(W)\eta(Y) + nB(W)\eta(Y).$$

Replacing $Y = \xi$ in (4.8) then using (2.1) and (2.6), we get

$$(4.9) \qquad (n-1)A(W) = nB(W).$$

Using (4.9) in (4.8), we obtain

$$(4.10) \qquad S(Y,\phi W) = -(n-1)g(Y,\phi W).$$

Again, replacing Y by ϕY both sides in the above equation (4.10) and using the equations (2.2) and (2.14), we obtain

$$S(Y,W) = -(n-1)g(Y,W),$$

i.e., the manifold is an Einstein manifold. □

5. Semi-Generalized Concircular ϕ-Recurrent P-Sasakian manifolds

Definition 5.1. A P-Sasakian manifold (M^n, g) is called semi-generalized concircular φ-recurrent if its concircular curvature tensor

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{n(n-1)}[g(Y,Z)X - g(X,Z)Y] \tag{5.1}$$

satisfies the condition

$$\varphi^2((\nabla_W C)(X,Y)Z) = A(W)C(X,Y)Z + B(W)g(Y,Z)X, \tag{5.2}$$

where A and B are defined as (4.2) and r is the scalar curvature of the manifold (M^n, g).

Theorem 5.1. *Let (M^n, g) be a semi-generalized concircular φ-recurrent P-Sasakian manifold then*

$$\left[-(n-1) - \frac{r}{n}\right]A(W) + nB(W) = 0.$$

Proof. Let us consider a semi-generalized φ-recurrent P-Sasakian manifold. Then by virtue of (2.1) and (5.2), we have

$$\begin{aligned}(\nabla_W C)(X,Y)Z &- \eta((\nabla_W C)(X,Y)Z)\xi \\ &= A(W)C(X,Y)Z + B(W)g(Y,Z)X,\end{aligned} \tag{5.3}$$

from which it follows that

$$\begin{aligned}g((\nabla_W C)(X,Y)Z, U) &- \eta((\nabla_W C)(X,Y)Z)\eta(U) \\ &= A(W)g(C(X,Y)Z, U) + B(W)g(Y,Z)g(X,U).\end{aligned} \tag{5.4}$$

Let $\{e_i\}, i = 1, 2, ...n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (5.4) and taking summation over i, $1 \le i \le n$, we get

$$\begin{aligned}(\nabla_W S)(X,U) &= \frac{W(r)}{n}g(X,U) - \frac{W(r)}{n}\eta(X)\eta(U) \\ &+ (\nabla_W S)(X,\xi)\eta(U) + nB(W)g(X,U) \\ &+ \left[S(X,U) - \frac{r}{n}g(X,U)\right]A(W).\end{aligned} \tag{5.5}$$

Replacing U by ξ in (5.5) and using (2.1) and (2.13), we have

$$\left[-(n-1) - \frac{r}{n}\right]A(W)\eta(X) + nB(W)\eta(X) = 0. \tag{5.6}$$

Putting $X = \xi$ in (5.6), we obtain

$$\left[-(n-1) - \frac{r}{n}\right]A(W) + nB(W) = 0.$$

This completes the proof. □

Theorem 5.2. *A semi-generalized concircular φ-recurrent P-Sasakian manifold is an Einstein manifold.*

Proof. Putting $X = U = e_i$ in (5.4) and taking summation over i, $1 \leq i \leq n$, we get

$$\begin{aligned}(\nabla_W S)(Y, Z) &= \sum_{i=1}^{n} g((\nabla_W R)(e_i, Y)Z, \xi)g(e_i, \xi)\\ &+ \frac{W(r)}{n} g(Y, Z) - \frac{W(r)}{n(n-1)} [g(Y, Z) - \eta(Y)\eta(Z)]\\ &+ \left[S(Y, Z) - \frac{r}{n} g(Y, Z)\right] A(W) + nB(W)g(Y, Z). \end{aligned} \tag{5.7}$$

Replacing Z by ξ in (5.7) and using (5.6), we have

$$(\nabla_W S)(Y, Z) = \frac{W(r)}{n}\eta(Y). \tag{5.8}$$

We know that

$$(\nabla_W S)(Y, \xi) = \nabla_W S(Y, \xi) - S(\nabla_W Y, \xi) - S(Y, \nabla_W \xi),$$

using (2.4), (2.5) and (2.13) in the above relation, it follows that

$$(\nabla_W S)(Y, \xi) = -(n-1)g(Y, \phi W) - S(Y, \phi W). \tag{5.9}$$

In view of (5.8) and (5.9), we obtain

$$S(Y, \phi W) = -(n-1)g(Y, \phi W) - \frac{W(r)}{n}\eta(Y). \tag{5.10}$$

Replacing Y by φY in (5.10) then using (2.2), (2.6) and (2.14), we obtain

$$S(Y, W) = -(n-1)g(Y, W).$$

□

6. Semi-generalized M-Projective φ-recurrent P-Sasakian manifolds

Definition 6.1. A P-Sasakian manifold (M^n, g) is called semi-generalized M-projective curvature tensor [10]

$$\begin{aligned} W^*(X, Y)Z &= R(X, Y)Z - \frac{1}{2(n-1)}[S(Y, Z)X - S(X, Z)Y\\ &+ g(Y, Z)QX - g(X, Z)QY]. \end{aligned} \tag{6.1}$$

satisfies the condition

$$\begin{aligned} &((\nabla_V W^*)(X, Y)Z) - \eta((\nabla_V W^*)(X, Y)Z)\xi\\ &= A(V)W^*(X, Y)Z + B(V)g(Y, Z)X, \end{aligned} \tag{6.2}$$

where A and B are defined as (4.2) and r is a scalar curvature of the manifold (M^n, g).

Theorem 6.1. *Let (M^n, g) be a semi-generalized M-Projective φ-recurrent P-Sasakian manifold then*

$$-\left[\frac{n^2 - n + r}{2(n-1)}\right] A(V) + nB(V) = 0.$$

Proof. Let us consider a semi-generalized φ-recurrent P-Sasakian manifold. Then by virtue of (2.1) and (6.2), we have

$$\begin{aligned} & (\nabla_V W^*)(X, Y)Z - \eta((\nabla_V W^*)(X, Y)Z)\xi \\ = \ & A(V)W^*(X, Y)Z + B(V)g(Y, Z)X, \end{aligned} \tag{6.3}$$

from which it follows that

$$\begin{aligned} & g((\nabla_V W^*)(X, Y)Z, U) - \eta((\nabla_V W^*)(X, Y)Z))\eta(U) \\ = \ & A(V)g(W^*(X, Y)Z, U) + B(V)g(Y, Z)g(X, U). \end{aligned} \tag{6.4}$$

Let $\{e_i\}, i = 1, 2, ...n$ be an orthonormal basis of the tangent space at any point of the manifold. Then putting $Y = Z = e_i$ in (6.4) and taking summation over i, $1 \leq i \leq n$, we get

$$\begin{aligned} & \frac{n}{2(n-1)} (\nabla_V S)(X, U) - \frac{1}{2(n-1)} V(r)g(X, U) \\ - & \frac{n}{2(n-1)} (\nabla_V S)(X, \xi)\eta(U) + \frac{1}{2(n-1)} V(r)\eta(X)\eta(U) \\ = & \left[\frac{n}{2(n-1)} S(X, U) - \frac{r}{2(n-1)} g(X, U)\right] A(V) \\ + & nB(V)g(X, U). \end{aligned} \tag{6.5}$$

Replacing U by ξ in (6.5) and using (2.1) and (2.6), we have

$$-A(V)\left[\frac{n^2 - n + r}{2(n-1)}\right] \eta(X) + nB(V)\eta(X) = 0. \tag{6.6}$$

Putting $X = \xi$ in (6.6), we obtain

$$-\left[\frac{n^2 - n + r}{2(n-1)}\right] A(V) + nB(V) = 0.$$

This completes the proof. □

Theorem 6.2. *A semi-generalized M-Projective φ-recurrent P-Sasakian manifold is an Einstein manifold.*

Proof. Putting $X = U = e_i$ in (6.4) and taking summation over $i, 1 \le i \le n$, we get

$$\begin{aligned}\frac{n}{2(n-1)}(\nabla_V S)(Y,Z) &= \sum_{i=1}^{n} g((\nabla_V R)(e_i,Y)Z,\xi)g(e_i,\xi)\\ &+ \frac{1}{2(n-1)}V(r)g(Y,Z)\\ &- \frac{1}{2(n-1)}[(\nabla_V S)(Y,Z)g(\xi,\xi)-(\nabla_V S)(\xi,Z)\eta(Y)\\ &+ g(Y,Z)(\nabla_V S)(\xi,\xi)-(\nabla_V S)(Y,\xi)\eta(Z)\ ]\\ &+ \left[\frac{n}{2(n-1)}S(Y,Z)-\frac{r}{2(n-1)}g(Y,Z)\right]A(V)\\ &+ nB(V)g(Y,Z). \end{aligned} \tag{6.7}$$

Replacing Z by ξ in (6.7) and using (2.1), (2.6) and (2.13), we have

$$(\nabla_V S)(Y,\xi) = -\frac{1}{n}V(r)\eta(Y). \tag{6.8}$$

We know that

$$(\nabla_V S)(Y,\xi) = \nabla_V S(Y,\xi) - S(\nabla_V Y,\xi) - S(Y,\nabla_V \xi),$$

using (2.4), (2.5) and (2.13) in above relation, it follows that

$$(\nabla_V S)(Y,\xi) = -(n-1)g(Y,\varphi V) - S(Y,\varphi V). \tag{6.9}$$

In view of (6.8) and (6.9)

$$S(Y,\varphi V) = -(n-1)g(Y,\varphi V) + \frac{1}{n}V(r)\eta(Y). \tag{6.10}$$

Replacing Y by φY in (6.10) then using (2.2) and (2.14), we get

$$S(Y,V) = -(n-1)g(Y,V).$$

This completes the proof. □

7. Three Dimensional Locally Semi-Generalized φ-recurrent P-Sasakian manifolds

Theorem 7.1. *The curvature tensor of three dimensional semi-generalized φ-recurrent P-Sasakian manifold is given by*

$$R(X,Y,Z) = \left[\frac{dr(e_i)}{2A(e_i)} - \frac{B(e_i)}{A(e_i)}\right] g(Y,Z)X - \frac{dr(e_i)}{2A(e_i)}\, g(X,Z)Y.$$

Proof. In a three-dimensional Riemannian manifold (M^3, g), we have

$$\begin{aligned} R(X,Y)Z &= g(Y,Z)QX - g(X,Z)QY + S(Y,Z)X \\ &- S(X,Z)Y + \frac{r}{2}\,[g(X,Z)Y - g(Y,Z)X], \end{aligned} \tag{7.1}$$

where Q is the Ricci operator, i.e., $S(X,Y) = g(QX,Y)$ and r is the scalar curvature of the manifold. Now putting $Z = \xi$ in (7.1) and using (2.13), we get

$$\begin{aligned} R(X,Y)\xi &= \eta(Y)QX - \eta(X)QY + (n-1)[\eta(X)Y - \eta(Y)X] \\ &+ \frac{r}{2}[\eta(X)Y - \eta(Y)X]. \end{aligned} \tag{7.2}$$

Using (2.9) in (7.2), we have

$$\left[(2-n) - \frac{r}{2}\right][\eta(X)Y - \eta(Y)X] = \eta(Y)QX - \eta(X)QY. \tag{7.3}$$

Putting $Y = \xi$ in the equation (7.3) and using the equations (2.6) and (2.12), we get

$$QX = \left[(3-2n) - \frac{r}{2}\right]\,\eta(X)\xi - \left[(2-n) - \frac{r}{2}\right]\,X. \tag{7.4}$$

Therefore, it follows from (7.4) that

$$S(X,Y) = \left[(3-2n) - \frac{r}{2}\right]\,\eta(X)\eta(Y) - \left[(2-n) - \frac{r}{2}\right]\,g(X,Y). \tag{7.5}$$

Thus from (7.1), (7.4) and (7.5), we get

$$\begin{aligned} R(X,Y)Z &= \left[2(2-n) - \frac{r}{2}\right][g(X,Z)Y - g(Y,Z)X] \\ &+ \left[(3-2n) - \frac{r}{2}\right][g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ &+ \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y]. \end{aligned} \tag{7.6}$$

Taking the covariant differentiation to the both sides of the equation (7.6), we get

$$\begin{aligned} (\nabla_W R)(X,Y)Z &= -\frac{dr(W)}{2}[g(X,Z)Y - g(Y,Z)X + g(Y,Z)\eta(X)\xi \\ &- g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ &+ \left[(3-2n) - \frac{r}{2}\right][g(Y,Z)\eta)(X) - g(X,Z)\eta)(Y)]\,(\nabla_W\xi) \\ &+ \left[(3-2n) - \frac{r}{2}\right][g(Y,Z)\xi - \eta(Z)Y]\,(\nabla_W\eta)(X) \\ &+ \left[(3-2n) - \frac{r}{2}\right][\eta(Y)X - \eta(X)Y]\,(\nabla_W\eta)(Z) \\ &- \left[(3-2n) - \frac{r}{2}\right][g(X,Z)\xi - \eta(Z)X]\,(\nabla_W\eta)(Y). \end{aligned} \tag{7.7}$$

Noting, that we may assume that all vector fields X, Y, Z, W are orthogonal to ξ and using (2.1), we get

$$\begin{aligned}(\nabla_W R)(X,Y)Z &= -\frac{dr(W)}{2}[g(X,Z)Y - g(Y,Z)X] \\ &+ \left[(3-2n) - \frac{r}{2}\right][g(Y,Z)(\nabla_W \eta)(X) \\ &- g(X,Z)(\nabla_W \eta)(Y)]\xi.\end{aligned} \tag{7.8}$$

Applying φ^2 to the both side of (7.8) and using (2.1) and (2.6), we get

$$\varphi^2((\nabla_W R)(X,Y)Z) = \frac{dr(W)}{2}\left[g(Y,Z)X - g(X,Z)Y\right]. \tag{7.9}$$

By (4.1), the equation (7.9) reduces to

$$A(W)R(X,Y)Z = \left[\frac{dr(W)}{2} - B(W)\right]g(Y,Z)X - \frac{dr(W)}{2}g(X,Z)Y.$$

Putting $W = \{e_i\}$, where i=1, 2, 3 is an orthonormal basis of the tangent space at any point of the manifold and taking summation over i, $1 \le i \le 3$, we obtain

$$R(X,Y)Z = \left[\frac{dr(e_i)}{2A(e_i)} - \frac{B(e_i)}{A(e_i)}\right] g(Y,Z)X - \frac{dr(e_i)}{2A(e_i)}\, g(X,Z)Y.$$

□

8. Conclusion

This paper is all about the study of geometrical properties of a semi generalized recurrent Para-Sasakian manifold. We prove that a semi generalized ϕ−recurrent Para-Sasakian manifold is an Einstein manifold. It is also stabilised that a semi generalized M projective ϕ−recurrent Para-Sasakian manifold and semi generalized concircular ϕ−recurrent Para-Sasakian manifolds are also Einstein manifold.

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